BULETINUL Universităţii Petrol – Gaze din Ploieşti	Vol. LVIII No. 2/2006	111 - 116	Seria Matematică - Informatică - Fizică

Visualization Techniques of the Material Fault Shape

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Abstract

The methods of emphasizing the material non-homogeneities of the material analyzed up to now, give directives about their occurring, without being able to provide clear information regarding their feature. Further a detection method is presented, called tomography taking, through which the fault geometry is emphasized.

Key words: *tomography, fault geometry, Fourier transformata*

Tomography Taking by Means of Elastic Waves Propagating in Straight Line

A very important thing which must be established from the beginning, it is the size order of a potential fault, which could be tomographied, by using the simplifying hypothesis of the diffraction non existence.

Considering that the elastic waves have speeds between 2000-6000 m/s in solids, an oscillation with a frequency of 1 MHz has a wavelength λ in the range 2-6 mm, that is in the frame of decimeters, or even more. By means with these statements one can present a mathematical method of tomographical reconstruction having as a base Fourier Slice Theorem, which can be announced: The monodimensional Fourier transformata of projection at θ angle of an object defined by distribution $f(x, y)$ is equal to bidimensional Fourier transformata of $f(x, y)$, on a radial section crossing the object at angle θ (Fig. 1).

Performing the Fourier's transformata for a large number of projections, at infinite limit, under angles varying from 0 to 360°, may be practically obtained the distribution bidimensional Fourier transformata $F_{2D}\{f(x, y)\}$, which by inversion will provide the function $f(x, y)$ which is in direct relation with the fault shape.

In order to prove the above mentioned theorem it is to start from the Fourier transformata definition 1D, 2D and of the projection under angle θ. In this way, the following expressions in Cartesian coordinates may be written:

$$(a) 1D\,FT : F(k_x) = \int_{-\infty}^{\infty} f(x) e^{-jk_x x} dx$$

$$(b)\,2D\,FT : F(k_x, k_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) e^{-j(k_x x + k_y y)} dx dy \,. \tag{1}$$

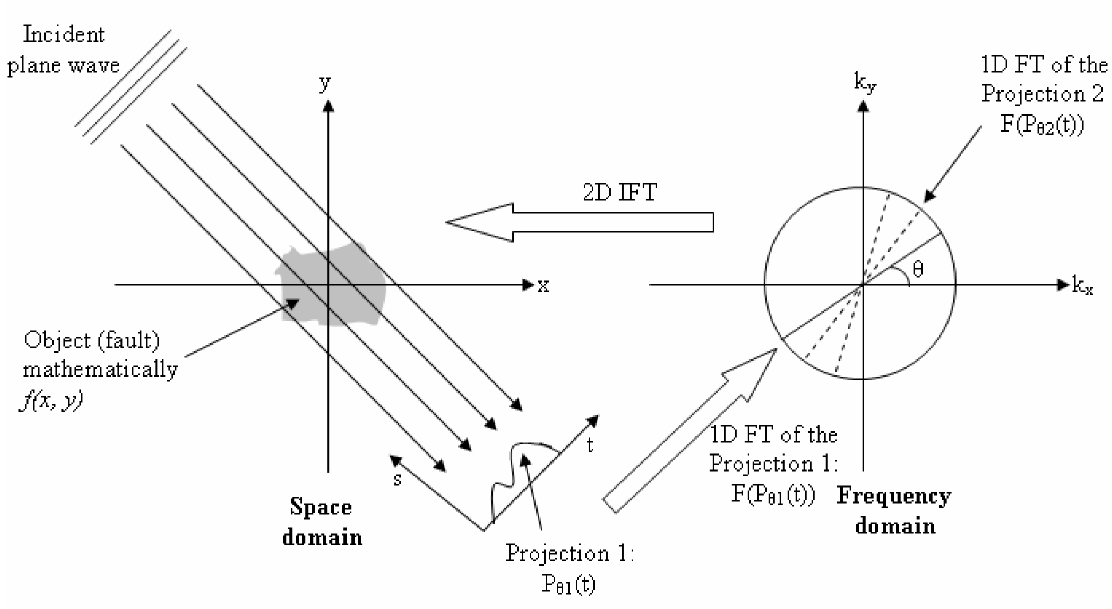

Fig. 1. Schematic description of the method of tomographic reconstruction of the projection faults

The inverse bidimensional transformata is written in the form:

$$2D\,IFT : F(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(k_x, k_y) e^{j(k_x x + k_y y)} dk_x dk_y \,. \tag{2}$$

In polar coordinates $F(k_x, k_y)$ is to be written in the form $F(k_r, \theta)$ where:

$$\begin{aligned} k_x &= k_r \cos\theta \\ k_y &= k_r \sin\theta \end{aligned} \,. \tag{3}$$

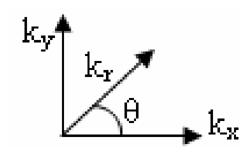

Fig. 2. Schematic description of (3)

It must be stated that the incident wave on the object is in interaction, with it, being under the influence of attenuations and reflexions on the propagation direction. In these conditions, the projection $P_\theta(t)$ is defined as being the line integrate along the direction s parallel to the propagation way (Fig. 1):

$$P_\theta(t) = \int_{-\infty}^{\infty} f(t,s) ds \,. \tag{4}$$

The link relations between (t,s) and (x,y) are:

$$\begin{bmatrix} t \\ s \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}. \tag{5}$$

t, s are both of the space coordinates, t being not in relation with the time:

Taking into account (3), (4) the Fourier transformata of $P_\theta(t)$ is:

$$F(P_\theta(t)) = S_\theta(k_r) = \int_{-\infty}^{\infty} P_\theta(t) e^{-jk_r t} dt = \int_{-\infty}^{\infty} f(t,s) e^{-jk_r t} dt \,. \tag{6}$$

Performing the variable changes from (s,t) in (x,y) the Fourier transformata Theorem of the section:

$$S_\theta(k_r) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-jk_r(x\cos\theta + y\sin\theta)} dxdy = F(k_r,\theta) \tag{7}$$

may be obtained.

In conclusions, performing $F\{P_\theta(t)\}$ for the angles $\theta_1, \theta_2 ... \theta_N$, $F(k_x,k_y)$ on radial line for the angles $\theta_2 ... \theta_N$ are obtained. If N is sufficiently large, an estimate of a high quality $\hat{F}(k_x,k_y)$, for $F(k_x,k_y)$, for any k_x, k_y may be obtained. By making an inversion of $\hat{F}(k_x,k_y)$ on the computer, by using the method IFFT, will reach $\hat{f}(x,y)$, and by it the identification process of the fault shape is closed.

The performing of a bidimensional Fourier transformata is an operation using large calculation resources, especially when large resolutions are wanted to be obtained. In this case, a method of alternative reconstruction which does not imply a bidimensional inversion will be presented.

It is initially supposed that $F(k_x,k_y)$ is accurately known. Performing the variable change $(k_x,k_y) \to (k_r,\theta)$ the expression (2) may be written as follows:

$$f(x,y) = \int_0^{2\pi}\int_0^{\infty} F(k_r,\theta) e^{jk_r(x\cos\theta + y\sin\theta)} k_r dk_r d\theta$$

$$f(x,y) = \int_0^{\pi}\int_0^{\infty} F(k_r,\theta) e^{jk_r(x\cos\theta + y\sin\theta)} k_r dk_r d\theta + \\ + \int_0^{\pi}\int_0^{\infty} F(k_r,\theta+\pi) e^{jk_r(x\cos(\theta+\pi) + y\sin(\theta+\pi))} k_r dk_r d\theta \,.$$

Because:

$$\begin{aligned} F(k_r,\theta+\pi) &= F(-k_r,\theta) \\ \cos(\theta+\pi) &= -\cos\theta \\ \sin(\theta+\pi) &= -\sin\theta \end{aligned} \tag{8}$$

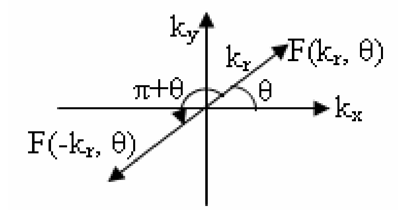

Fig. 3. Schematic description of (8)

$$\begin{aligned}&\int_0^{\pi}\int_0^{\infty} F(k_r,\theta+\pi)e^{jk_r(x\cos(\theta+\pi)+y\sin(\theta+\pi))}k_r dk_r d\theta = \\ &= \int_0^{\pi}\int_{-\infty}^{\infty} F(k_r,\theta)e^{jk_r(x\cos(\theta+\pi)+y\sin(\theta+\pi))}(-k_r)dk_r d\theta\end{aligned} \quad . \tag{9}$$

Therefore:

$$\begin{aligned}&f(x,y) = \int_0^{\pi}\int_{-\infty}^{\infty} F(k_r,\theta)e^{jk_r(x\cos\theta+y\sin\theta)}\,|\,k_r\,|\,dk_r d\theta = \\ &= \int_0^{\pi}\left[\int_{-\infty}^{\infty} S_\theta(k_r)e^{jk_r t}\,|\,k_r\,|\,dk_r\right]d\theta\end{aligned} \quad . \tag{10}$$

$Q_\theta(t)$ is not even $P_\theta(t)$, that is, the known projection from physical measurements may be written as a convolution between $P_\theta(t)$ and a function $h(t)$.

$$Q_\theta(t) = P_\theta(t) * h(t)\,, \tag{11}$$

where $h(t)$ is given by

$$\int_{-\infty}^{\infty} h(t)e^{-jk_r t}dt = |\,k_r\,| \text{ sau } h(t) = \int_{-\infty}^{\infty} |\,k_r\,|\,e^{jk_r t}dk_r\,. \tag{12}$$

Two means for obtaining $Q_\theta(t)$ by using the computer may be distinguished:

1. It is $h(t)$ calculated, the convolution $P_\theta(t) * h(t)$ is carried out and then $f(x,y)$ is obtained as being $\int_0^k [P_0(t) * h(t)]d\theta$ wares.
2. It is $F\{P_0(t)\}$ calculated, after that $f(x,y)$ is obtained, as being $\int_0^{\pi} F^{-1}\{F(P_\theta(t))|\,k_r\,|\}d\theta'$ where $t = x\cos\theta + y\sin\theta, k_x = k_r\cos\theta, k_y = k_r\sin\theta$.

From point of view of the calculation effort, it is more efficient to carry out the convolution of two functions $f(t)$ and $g(t)$ by multiplying the rapide Fourier transformata and carrying out the result inversion than by using the formula $f(t) * g(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau$, therefore the second method is preferred. The calculation algorithms of distribution $f(x,y)$ from projections may be summed up in the following way:

1. $f(x, y) = 0$ is setted, anyone might be x, y
2. the projection $P_{\theta 1}(t)$ is measured
3. the rapide Fourier transformata of the projection $P_{\theta 1}(t)$ is carried out
4. the product $F\{P_{\theta 1}(t)\} | k_r |$ is carried out
5. the rapid inverse Fourier transformata of the product from the previously point is calculated, the obtained result being a section of $f(x, y)$ at the angle θ_1
6. the steps 2, 3, 4, 5 are repeated for the projections under the angles $\theta_1, \theta_2, \theta_3, ..., \theta_n \in [0...380^0]$.

Taking Tomography by Means of Elastic Waves Taking Into Account the Diffraction Phenomenon

In the previously paragraph developed theory does not take into account the diffraction, an extremely obvious phenomenon in the case of the elastic waves interaction with different bodies. When the size of the material fault becomes comparable to the wavelength, its projection cannot be presented as a line integral (formula 4). Another method of emphasizing a mathematical contact (link) between a fault projection and its physical proprieties must be found.

Generally, the interaction between an object and an elastic disturbance is shaped by an equation with partial derivates. In the case of gaseous or liquid media including only gaseous or liquid objects, this is obtained by eliminating the pressure from the system (Find Refference). In this way, may be obtained a propagation equation, called as acoustic equation, having the shape:

$$\nabla^2 u(r,t) - \frac{1}{c^2(r)} \frac{\partial^2 u(r,t)}{\partial t^2} = 0, \tag{13}$$

where u is the moving from the equilibrium position, and c, the disturbance propagation speed.

For the general situation where the elastic wave crosses a mixture of bodies being in different aggregation states, the interaction equation is given by the expression (Find Refference) or (Find Refference), which, as it has been shown, will had generally to the appearance of two elastic waves propagating with different speeds. However, in thin solid plates and on the separation surface of a semi-infinite solid medium only one important wave appears, and consequently may be supposed that this checks up only an equation being of acoustic type. Further only this equation will be used, because it is assumed that the present work is going to analyze the detection by tomography only of the faults situated on the surface of a solid, or in its close vicinity comparatively to the wave length on the elastic radiation for survey.

Starting from (13) it is proved that there is a mathematical method analogous to Fourier's Theorem of the Section, called Fourier's Theorem of Diffraction, which states that The monodimensional Fourier transformata of projection to angle θ of an definite object by $f(x, y)$ distribution is equal to bidimensional Fourier transformata of $f(x, y)$ on a semi-circular section crossing the object. In order to prove this theorem, in (13) a solution having the shape (14) it is tried

$$u(r,t) = U \exp[i(\omega t - k(r)r] = U \exp[-ik(r)r] \exp[i\omega t] = u(r) \exp[i\omega t]. \tag{14}$$

In this way the following a temporal equation is obtained

$$\left(\nabla^2 + k^2(r)\right)u(r) = 0\,, \tag{15}$$

where $k(r) = \omega / c(r)$ is the space frequency and ω the temporal frequency.

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Tehnici de vizualizare a formei defectelor de material

Rezumat

Tomografierea este o modalitate de calculare a formei unui obiect pornindu-se de la proiecțiile generate prin "iluminarea" sa sub diverse unghiuri cu o radiație oarecare, ce poate fi de natură electromagnetică sau elastică. În cazul în care radiația folosită are p lungime de undă mult mai mică decât dimensiunile corpului studiat metoda matematică de reconstrucție pe baza cunoașterii proiecțiilor e mai simplă, neînregistrându-se fenomene de difracție. Pentru lungimi de undă mari, cazul undelor mecanice, difracția trebuie luată în considerație în majoritatea situațiilor.